ISRA 2019 AMSTERDAM

PROCEEDINGS of the
International Symposium on Room Acoustics

15 to 17 September 2019 in Amsterdam, Netherlands

NAG NEDERLANDS AKOESTISCH GENOOTSCHAP

Finding the resonance frequencies of non-shoebox shaped rooms

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ABSTRACT

To find the resonance frequencies of a non-shoebox shaped room is a classical problem in room acoustics, which is straightforward to solve with the finite element method. Still, the finite element method requires specialized software which is often quite complex to use. Here, we explore two other approaches to study this problem. A first is to use a type of waveguide modeling which allows a changing waveguide cross-section, the so-called Mode Matching Method, which has successfully been used for modeling horn loudspeakers. This approach can efficiently model quite general room geometries, as long as cross-sections along one dimensions are rectangular, [Kolbrek & Svensson, 140th AES Conv., paper no. 9506, 2016]. A second approach is to use a shoebox to create an outer "bounding box" for the room of interest, and then introduce secondary sources of dipole type at the locations of walls inside this shoebox to fulfill the boundary conditions of the walls. Possibilities and limitations with these approaches will be presented.

Keywords: Room resonances

1. INTRODUCTION

Most textbooks on room acoustics describe the case of a shoebox-shaped room with (almost) rigid walls, for which the eigenfunctions and eigenvalues (resonance frequencies) can be derived easily. This theoretical case is quite useful in practice since most rooms are built with a shape which is close to the shoe-box, and walls are typically quite hard. A large number of simple tools are available, online and otherwise, for finding these resonance frequencies, and compute the frequency response of a source in such a room. However, in many practical cases, a room often does not fit this model well. Theoretical solutions for non-rigid boundary conditions (but constant boundary conditions across each wall) are available, (1), but they have been much less common than the simplified rigid-wall solution, with the well-known "cos-cos-cos" mode functions. The mathematics of these general solutions might be a bit daunting, and the resonance frequencies can not be found with an explicit formula but must be found from the numerical solution of equations with complex-valued coefficients and roots. A small number of point impedances, in otherwise rigid walls, can be handled by introducing secondary sources in the rigid-wall solution and this approach could be used to efficiently study the effect of a small number of Helmholtz absorbers, (2). General solution methods such as the Finite Element Method (FEM), or Finite Differences, can certainly be used, and can predict sound fields accurately, even if it turns out that finding the right boundary conditions can be very challenging, (3).

Apart from the impedance of the boundaries, the geometrical shape of the room is a second factor for which it could be practical with more general solutions than the classical shoe-box shaped room. Again, the general FEM can be used, and finding the resonance frequencies/eigenvalues is a task which is efficiently handled by the FEM. The FEM is a very powerful and general tool, and the softwares might be expensive and demanding for the user. Therefore, in this paper we explore alternative ways to find the resonance frequencies of non-shoebox shaped rooms. The special case of a shoe-box shaped room with one end wall tilted, a trapezoidal room shape, has been studied in a number of papers, using variants of a method that has been called the Green's function method. With that method, the known eigenfunctions for a larger shoe-box shaped bounding box are used to express the unknown eigenfunctions of the smaller irregularly shaped room, (4)-(6). The method uses a discretization of the irregularly shaped walls and sets up a matrix equation which can give the eigenvalues and eigenfunctions in an efficient way.

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Here, other approaches will be explored. A first approach was presented in (7), where it was shown that many room shapes can be modeled with the mode-matching method (MMM), which models the room as a waveguide with varying cross-section. Thus, if a room can be modeled as a "stack" of rectangularly shaped sections, then this approach can be an efficient method. This approach had been presented earlier, (8), for rooms with two symmetry axes. A second approach is to introduce secondary sources, or equivalent sources, which, in addition to a primary source, aim at fulfilling the boundary condition at walls that do not follow the shoebox shape.

These two approaches are presented in section 2, together with expressions for the eigenvalues of some triangular shapes, and the Green's function technique. In section 3, numerical examples are presented, together with some reproduced results from the previous study of the mode-matching method, (7), and from a study of trapezoidal rooms, (6).

2. THEORY AND METHODS

2.1 Eigenvalues

The mathematical task at hand is to find solutions to the Helmholtz equation,

$$\nabla^2 p + k^2 p = 0 \tag{1}$$

where p is the sound pressure field, for a wavenumber $k = \omega/c$, where c is the speed of sound , $\omega = 2\pi f$ and f is the frequency of the (harmonic) sound field. A Neumann boundary condition (BC), $\partial p/\partial n = 0$, is assumed here for all room boundaries, that is, ideally rigid walls. The (non-trivial) solutions to Eq. (1), under the Neumann BC, are the eigenvalues, k_n, which give the resonance frequencies, $f_n = ck_n/(2\pi)$. The resonance frequencies are needed in modal expressions for the sound field inside a room, but they also have the practical significance that the sound pressure amplitude can be particularly high at these frequencies, and consequently cause acoustic problems. Thus it might be interesting, per se, to find the eigenfrequencies, as part of the larger problem of describing the sound field. The modeshapes, or eigenfunctions, are also useful in practice, since, e.g., the placement of resonance absorbers, or source positions, or listening positions will depend heavily on the mode shapes. The eigenfrequencies for the shoebox-shaped room of size $l_x \cdot l_y \cdot l_z$, are well-known, (1),

$$f_{mnq} = \frac{c}{2}\sqrt{\left(\frac{m}{l_x}\right)^2 + \left(\frac{n}{l_y}\right)^2 + \left(\frac{q}{l_z}\right)^2} \tag{2}$$

where $m, n, q \in [0, 1, 2, ...]$. Very few geometries have explicitly known eigenfrequencies, especially shapes that are useful in practice. Somewhat useful are the cylindrical, (1) and spherical shapes. Here, we will explore the less well-known analytical solutions for triangular shapes. Triangular shapes are not very relevant for room acoustic cases, but serve as useful reference cases. Also, the triangular solutions can be expanded into three-dimensional cases by introducing a floor and a parallel ceiling. The equilateral triangle with all three sides having the sidelength a, and the rigid (Neumann) boundary condition has the two-dimensional eigenfrequencies, (9),

$$f_{mn} = \frac{2c}{3a}\sqrt{m^2 + mn + n^2}, \qquad m, n \in [0, 1, 2, ...] \tag{3}$$

Also the hemi-equilateral triangle, which is the equilateral triangle cut in two, and thus with sidelengths $a, a/2$, and $a\sqrt{3}/2$, and angles 90, 60, 30 degrees, has the eigenfrequencies in Eq. (3). A second triangle with known eigenfrequencies is the right isosceles triangle, with angles 90, 45, 45 degrees, and sidelengths a, a and $a\sqrt{2}$, which has the eigenfrequencies, (10),

$$f_{mn} = \frac{c}{2a}\sqrt{(m+n)^2 + n^2}, \qquad m, n \in [0, 1, 2, ...] \tag{4}$$

These resonance frequencies will be the same ones as those appearing for a square domain of size $a \cdot a$. Eigenfunctions for the latter triangle were given also in (1).

2.2 The mode-matching method (MMM)

The sound field in a waveguide, a straight duct with rigid walls, expanding in the z-direction, with a rectangular cross-section of size $l_x \cdot l_y$, for which the set of mode functions, $\varphi_{mn}(x, y)$, is known, can be described as

$$p(x, y, z) = \sum_{m,n=0}^{\infty} \left(A_{mn}\mathrm{e}^{-jk_z z} + B_{mn}\mathrm{e}^{jk_z z}\right)\varphi_{mn}(x, y) \tag{5}$$

where a time-harmonic factor $e^{j\omega t}$ has been suppressed, the mode functions are

$$\varphi_{mn}(x,y) = \cos\frac{m\pi x}{l_x}\cos\frac{n\pi y}{l_y} \tag{6}$$

and

$$k_z = \begin{cases} -\sqrt{k^2 - k_x^2 - k_y^2} & \text{if } k^2 < k_x^2 + k_y^2 \\ \sqrt{k^2 - k_x^2 - k_y^2} & \text{if } k^2 > k_x^2 + k_y^2 \end{cases} \tag{7}$$

The mode coefficient sets A_{mn} and B_{mn} can be found by applying the boundary conditions (BC) at the two ends of the duct section. Now, if two such ducts of different cross-sections are connected together, at the dashed line in Figure 1, then there are two sets of mode coefficients, $A^I_{m,n}$ and $B^I_{m,n}$, for the domain I, to the left of the dashed line, and two sets for the other domain to the right of the dashed line, $A^{II}_{m,n}$ and $B^{II}_{m,n}$. Since the sound fields must match at the interface, the mode coefficient sets can be related to each other, and a matrix expression can be formulated for each such waveguide step-change. This interconnection can be carried out step-wise, which forms the mode-matching method, and details can be found, e.g., in (11). For this method, the end section typically has a radiation impedance as boundary condition. However, for the study of rooms, as in this paper, the end section BC should be that of a rigid wall.

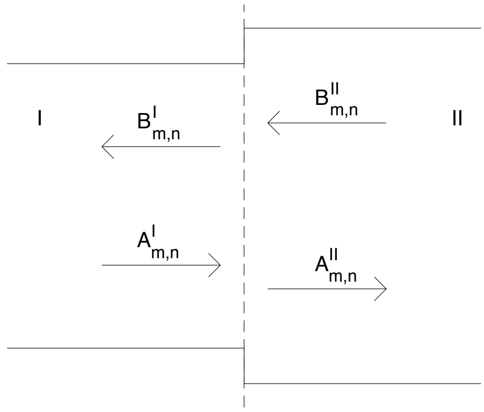

Figure 1 – An interface between two waveguide sections; the building block of the mode matching method.

2.3 The Green's function method

A number of studies are based on the method that a set of eigenfunctions and eigenvalues of a cavity of irregular shape can be expressed in terms of mode functions for a shoebox shaped "bounding box" for the cavity of interest, (4)-(6), as illustrated in Figure 2 for a trapezoidally shaped room with one tilted end wall. If the known mode functions for the bounding box are denoted $\varphi^{\mathrm{BB}}_{mnq}$, then the sound field in the irregularly shaped room is described in terms of these,

$$p(x,y,z) = \sum_{m,n,q=0}^{\infty} b_{mnq}\varphi^{\mathrm{BB}}_{mnq}(x,y,z) \tag{8}$$

with unknown mode coefficients b_{mnq}. As described in (4),(5), the eigenvalue problem described by Eq. (1) can, via an application of the Helmholtz integral, lead to the equation

$$(k^2 - k^2_{rst})b_{rst} = \sum_{m,n,q} b_{mnq}n_{rst,mnq} \tag{9}$$

where, for a case with the Neumann boundary condition (rigid walls), the terms

$$n_{rst,mnq} = \int\int_{S_{irregular}} \varphi_{mnq}\frac{\partial\varphi_{rst}}{\partial n}dS \tag{10}$$

are values that describe the coupling between mode function number *mnq* and the derivative of the mode function number *rst*. The integration should be done for the entire irregular cavity, but since the derivative of the mode function is zero everywhere that the irregular room's walls coincide with the bounding box, only the irregular part of the irregular box needs to be integrated across, the "Irregular wall" in Figure 2. Eq. (9) can be written as a matrix equation on a form which directly can give the eigenvalues, k^2, of the matrix equation.

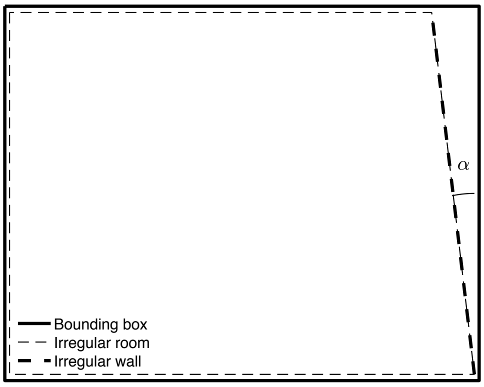

Figure 2 – A trapezoidal room, “Irregular room”, with one tilted end wall, “Irregular wall” , and its bounding box. The irregular room has been shrunk somewhat for illustration purposes.

2.4 Secondary sources

A number of methods are based on similar ideas: a set of sources, either monopoles or multipoles, have their amplitudes adjusted such that their combined sound fields fulfill some specified boundary condition. Scattering problems can thus be studied by removing the actual scattering object, and let a set of free-field radiating secondary sources, positioned inside the boundaries of the scattering object to be simulated, fulfill the boundary condition of the scattering object, together with an external primary source which ensonifies the object. Likewise, vibrating structures can be represented by such secondary sources. Several different names have been used in the literature: the equivalent source method, the method of auxiliary sources, the source simulation technique, the method of fundamental solutions, etc. (12).

For internal problems, the equivalent source method has been used for, e.g., studying non-rigid boundary conditions in shoebox-shaped rooms, (2), and the presence of scattering objects in shoebox-shaped rooms, (13). Quite generally, the problem can be formulated directly as a matrix equation, whereby a set of equivalent sources with unknown amplitudes Q_i have associated transfer functions, $H_{j,i}$ to the relevant field quantity at discrete boundary points, x_j, and also transfer functions, $H_{j,0}$, from the primary source with a known source amplitude Q_0. Thus, by storing the field quantity of interest at the boundaries, e.g., $\partial p_j/\partial n$, in a vertical vector **f**, the unknown equivalent source amplitudes in **q**, the transfer functions from equivalent sources to the boundary points in the matrix $\mathbf{H}_{B,ES}$, and the transfer functions from the original source to the boundary points in the vertical vector $\mathbf{h}_{B,0}$, the matrix equation can be written as

$$\mathbf{f} = \mathbf{h}_{B,0}Q_0 + \mathbf{H}_{B,ES}\mathbf{q}_{ES} \tag{11}$$

If the BC is that **f** should be zero, $\mathbf{q}_{ES}$ can be found via an inversion of the matrix $\mathbf{H}_{B,ES}$. Depending on the number of equivalent sources vs. the number of control points at the boundaries, various solution approaches need to be applied. Furthermore, the transfer functions in the **H**-matrices could in principle be free-field transfer functions, or transfer functions that fulfill parts of the BC of the problem. This latter case applies if one chooses transfer functions for a larger bounding box shoebox room, as described above for the Green’s function method. However, a direct application of the modal solution in Eq. (8) leads to that each element in the **H**-matrices will require a full modal summation, which becomes very computationally expensive. This fact also limited the method in (2), where equivalent sources, or “virtual pistons”, where introduced to simulate point impedances in a room with otherwise rigid walls. The transfer functions to compute were modal-sum expressions like in Eq. (8), and the number of transfer functions to be computed grew with the square of the number of boundary points with non-rigid boundary conditions. Therefore, the equivalent source approach is simply not very efficient in the form where a modal sum is used as transfer function.

3. NUMERICAL EXAMPLES

3.1 Validation of the mode-matching method

The mode-matching method applied to enclosed waveguides, i.e., cavities, was evaluated in (7), where the computed frequency responses were compared with an available analytical result for a shoebox case, and a finite-element calculation for one example geometry. Figure 3(a) shows the example geometry, with a truncated-wedge shaped floor plan. The mode-matching method was furthermore applied in three different directions, as is indicated in the inset figure in Figure 3(a). A full three-dimensional room was studied, with a constant room height of 2.4 m.

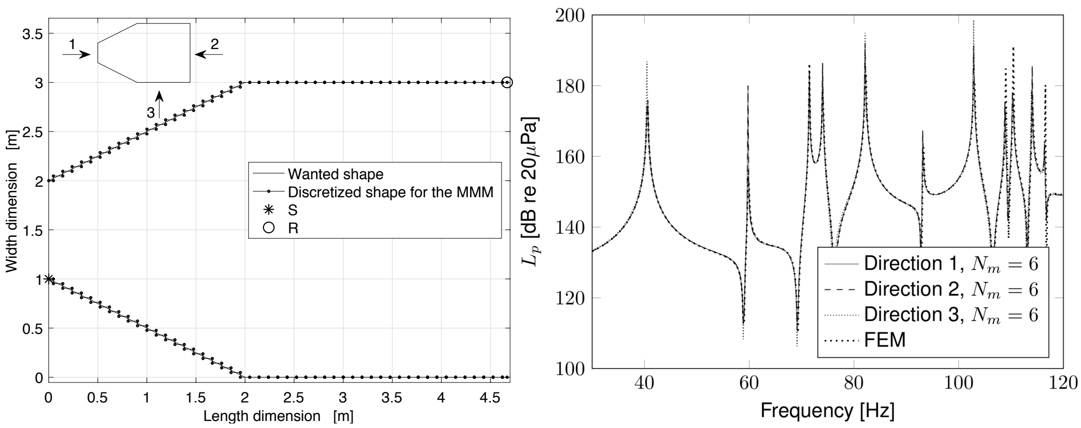

Figure 3 – (a) Floor plan of an example room studied in (7), with a source and a receiver position marked. The truncated-wedge shaped room was discretized into rectangular sections for the mode-matching method. The inset indicates three different propagation directions that can be chosen for the application of the method. (b) Frequency responses computed with the MMM as well as with the FEM.

The frequency responses computed with the MMM, as well as with the finite element method (FEM), are shown in Figure 3(b). For that study, 6 modes in each direction were used for each section in the MMM. The Comsol Multiphysics® software was used for the FEM, with a maximum mesh size of 0.45 m. For more details and results, see (7), where it was found that the MMM results agreed very well with the FEM results.

3.2 Resonance frequencies in triangular rooms, with the mode-matching method

In this section, the resonance frequencies in rooms with triangular floor plans are studied, using the mode matching method. The wedge angle of the triangle-shaped rooms is varied between 30 degrees and 45 degrees since reference results are available for these two endpoint values. The triangular room is represented by discretized sections with rectangular cross-sections. One example is shown in Figure 4(a), where the wanted shape is shown together with a discretized version with 20 such sections. Since the resonance frequencies will be determined from the peaks of a frequency response function, source and receiver positions are marked.

The speed of sound is set to 343.4 m/s and the floor area is arbitrarily set to 50 m^2. The mode-matching method requires a rectangular section at the end, so the triangles were discretized into 20 sections, with a length of 0.4 m cut off at the sharp end of the triangle. The number of modes was 6, and 1000 frequencies were evenly distributed between 10 and 70 Hz, thereby giving an uncertainty of ±0.03 Hz for the peak detection. Peaks in the frequency response were considered as resonance frequency candidates, see Figure 4(b), but a minimal “contrast” of a factor 1.25, between a peak value and the nearest response amplitude neighbours, was required for a peak to be considered as a peak. This value was found to suppress some spurious maxima for the chosen frequency resolution.

Figure 5 shows the nine lowest resonance frequencies computed accordingly, as function of wedge angle. As can be seen, the found values agree very well with the theoretical values for wedge angles 30 and 45 degrees, as given by Eqs. (3) and (4). Interestingly, the resonance frequencies vary quite little within the studied angle range. For these studied cases, the mode-matching method performs very well, and with a low maximum mode number of 6, 20 waveguide sections, and 1000 frequency values, the frequency response is computed within just a few seconds with a Matlab implementation on a laptop computer. It could be remarked that quite arbitrary room shapes can be studied for similar computational efforts.

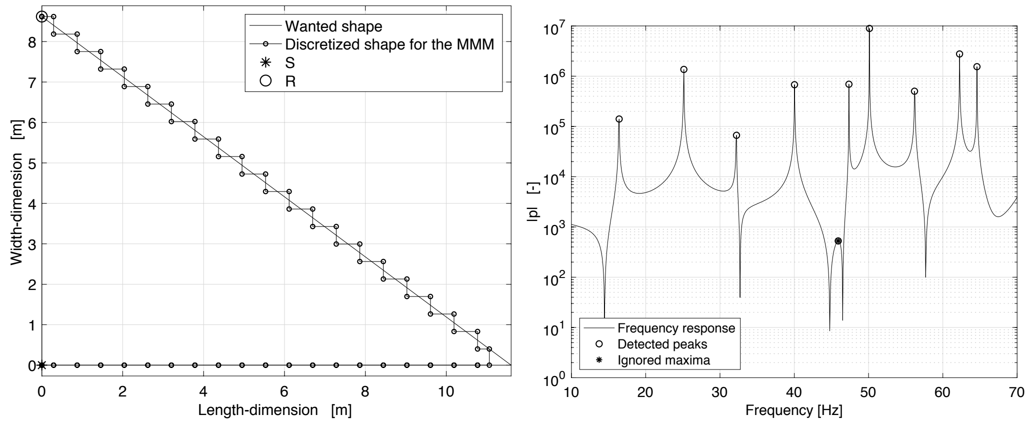

Figure 4 – (a) The floor-plan of a triangular room, represented by the wanted shape, and a discretized version with 20 sections, for the mode-matching method (MMM), with a source and receiver marked by a '*' and a circle, respectively. The floor area is 50 m^2 and the wedge angle (at the sharp end) is 36.6 degrees. (b) The frequency response for the triangular room in (a), as computed with the MMM. Peaks marked with circles were considered as resonance frequency candidates whereas the local response maximum marked with an asterisk was ignored.

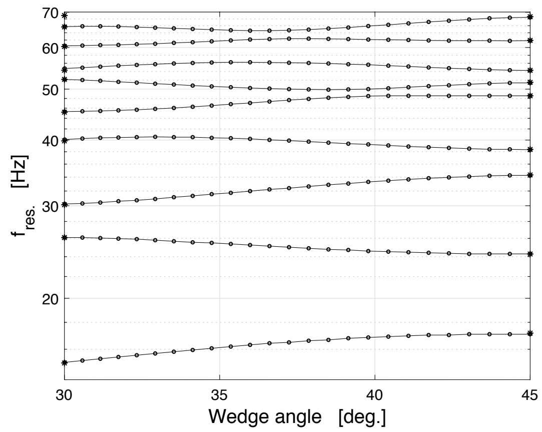

Figure 5 – The first nine resonance frequencies in a room with a right-angled triangular floor shape, as function of wedge angle, computed with the mode-matching method. The data points marked with asterisks, for angles 30 and 45 degrees, are theoretical values according to Eqs. (3) and (4). The floor area was 50 m^2 and the ceiling height low enough to make the room effectively two-dimensional.

3.3 Resonance frequencies in a trapezoidal room

In this section, trapezoidal rooms as in Figure 2 are studied. Such cases were studied with the Green's function method, and developments thereof, in (4)-(6). One case from (6) were investigated here: small rooms of width 0.575m and the average length 0.434m but with a tilted end wall. The third dimension was set very small here to remove its influence, whereas in ref. (6), the third dimension was 0.5 m, so that modes involving the third dimension were studied as well. The frequency response was computed with the MMM, and resonance frequency candidates were detected as peaks in the frequency response curves, as in Figure 4(b). Two perpendicular propagation directions were studied with the MMM, to ensure that the same resonance peak frequencies were detected. Two particular resonance frequencies, as function of wall angle, α, are plotted in Figure 6, together with the results from (6). As discussed in (5)-(6), a trapezoidal room of angle 0 is a shoebox-shaped room with well-defined resonance frequencies, and the resonance frequencies will change smoothly from these, as the wall angle is increased. Therefore, one can follow how one particular resonance frequency evolves as function of wall angle. The resonance frequencies in Figure 6 start as the (1,1) mode for $\alpha = 0$, in (a), and the (2,1) mode for $\alpha = 0$, in (b). For the results in Figure 6 (a), the average difference is 1.2% and the maximum is 2.8%, and for the results in Figure 6(b), the average difference is 0.8% and the maximum difference is 2.5%. For the MMM results here, a discretization of 50 sections was used, and

12 modes for each section. Further work is needed to investigate the effect of the discretization into sections and the number of modes on the accuracy of the mode matching method when applied to enclosed cavities like here. The computational aspects for the method's application to horn loudspeakers have been studied thoroughly, (11).

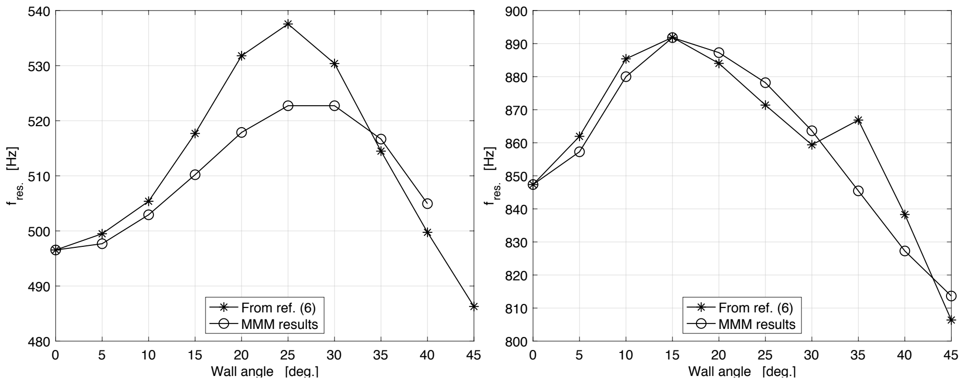

Figure 6 – The evolution of two different resonance frequencies in a trapezoidal room, as function of wall angle, computed with the mode-matching method and taken from ref. (6). Note the different ranges of the resonance frequency scales.

4. CONCLUSIONS

The mode matching method has been applied to the study of irregularly shaped rooms that can be modeled as waveguides, i.e., stacks of rectangular sections with varying areas. It gives an easy-to-use and efficient method for computing frequency responses in rooms with rigid walls, and the results have been found to agree very well with FEM results in a previous study, with theoretical results for two different triangularly shaped rooms, and reasonably well with results based on the developing the sound field in terms of modes for a rectangular bounding box.

The method of equivalent sources, employing a direct formulation which uses the mode functions of a rectangular bounding box was not employed any further because of inefficient computational aspects.

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